# Asymptotic homogenization for multiscale composites. Theoretical issues and applications to aged bone

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## Abstract

We discuss recent theoretical and computational advances [1, 2] in the context of multiscale composites, and their application to the bone hierarchical structure. Recent experimental data revealed a stiffening of aged cortical bone tissue, which could not be explained by common multiscale elastic material models. We explain these data by incorporating the role of mineral fusion via a new hierarchical modeling approach exploiting the asymptotic (periodic) homogenization (AH) technique for three-dimensional linear elastic composites. We quantify for the first time the stiffening that is obtained by considering a fused mineral structure in a softer matrix in comparison with a composite having non-fused cubic mineral inclusions. We integrate the AH approach in the Eshelby-based hierarchical mineralized turkey leg tendon model [3], which can be considered as a base for musculoskeletal mineralized tissue modeling. We model the finest scale compartments, i.e. the extrafibrillar space and the mineralized collagen fibril, by replacing the self-consistent scheme with our AH approach. This way, we perform a parametric analysis at increasing mineral volume fraction, by varying the amount of mineral that is fusing in the axial and transverse tissue directions in both compartments. Our effective stiffness results are in good agreement with those reported for aged human radius and support the argument that the axial stiffening in aged bone tissue is caused by the formation of a continuous mineral foam. Moreover, the proposed theoretical and computational approach supports the design of biomimetic materials which require an overall composite stiffening without increasing the amount of the reinforcing material [4].

## References

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