Topics in Continuum Mechanics applied to Biology/3:

Modeling skeletal muscle tissue

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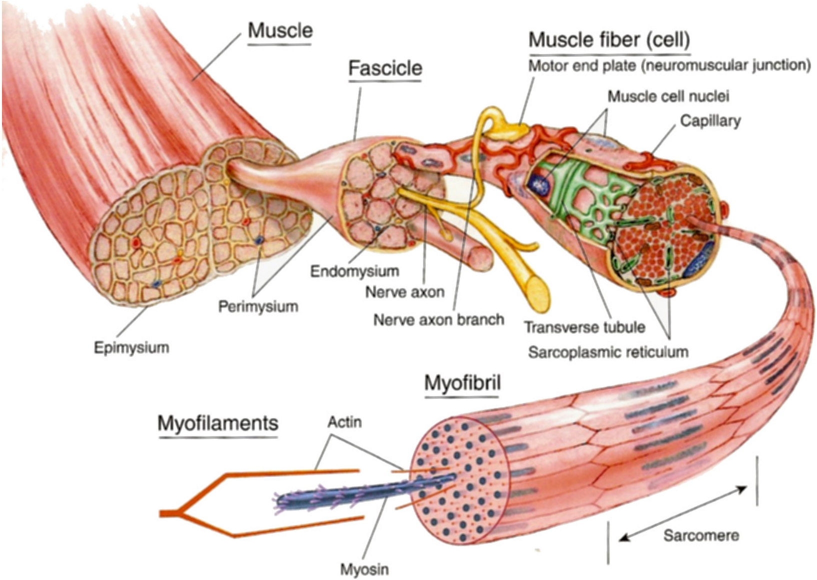
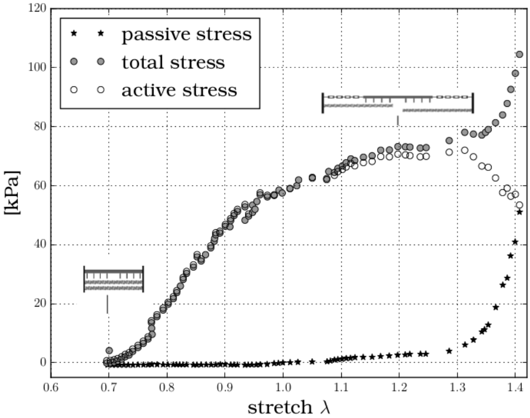
Skeletal muscle hierarchy

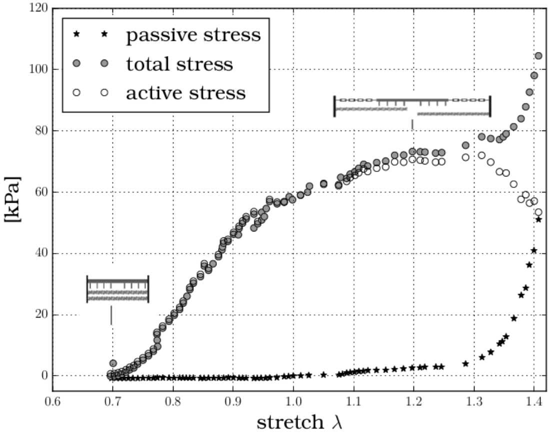


Some experimental data *in vivo* (rat tibialis anterior)



Hawkins & Bey, *A Comprehensive Approach for Studying Muscle-Tendon Mechanics*, J. Biomech. Eng., 116 (1994)

Active contraction



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- hyperelastic (at least in some regimes)

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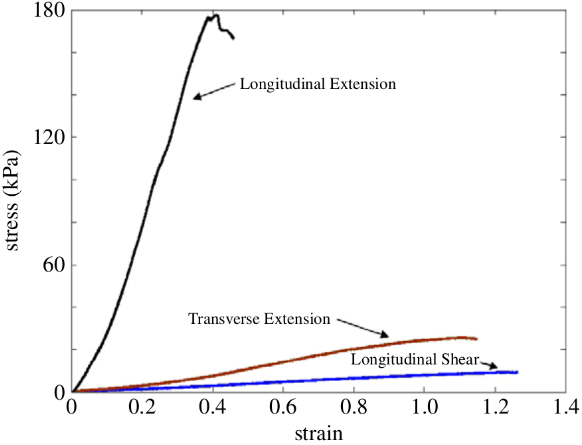

Morrow *et al.*, *Transversely isotropic tensile material properties of skeletal muscle tissue*, Journal of the Mechanical Behavior of Biomedical Materials, 3 (2010)

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- anisotropic (locally transversely isotropic)
- (almost) incompressible
- active/passive material
- activation is not an external parameter (unlike heart muscle tissue)

A widespread hyperelastic energy for biological tissues

Biological soft tissues are generally inelastic. They usually show hysteresis phenomena and viscoelastic behavior.
However, at least in some case, it is possible to drastically reduce the viscoelastic nonlinear constitutive prescription of a biological tissue to a hyperelastic one.

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A general feature of soft tissues in simple elongations is the following:

> *the slope of the stress-strain curve is approximately proportional to the tensile stress.*

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Such models can describe:

- muscle tissue
- arterial walls
- lung tissue
- visceral pleura
- visceral pericardium
- ...

Which exponential model?

Polynomial models	
Raghavan and Vorp	$W = c_1(I_1 - 3) + c_2(I_1 - 3)^2$
Knowles	$W = \frac{c_1}{2c_2}\left[\left(1 + \frac{c_2}{c_3}(I_1 - 3)\right)^{c_3} - 1\right]$
Exponential model	
Demiray	$W = \frac{c_1}{c_2}\left\{\exp\left[\frac{c_2}{2}(I_1 - 3)\right] - 1\right\}$
Demiray et al.	$W = \frac{c_1}{c_2}\left\{\exp\left[\frac{c_2}{2}(I_1 - 3)^2\right] - 1\right\}$
Holmes and Wow	$W = c_0\left(\exp\left(c_1(I_1 - 3)\right) + \exp\left(c_2(I_2 - 3)\right)\right) - c_0$
Arnoux et al.	$W = c_1 \exp\left(c_2(I_1 - 3)\right) - \frac{c_1 c_2}{2}(I_2 - 3)$
Singh et al.	$W = \frac{c_1}{2c_2} \exp\left(c_2(I_1 - 3) - 1\right) + \frac{c_3}{2}(I_2 - 3)^2$
Volokh and Vorp	$W = c_1 - c_1 \exp\left[-\frac{c_2}{c_1}(I_1 - 3) - \frac{c_3}{c_1}(I_1 - 3)^2\right]$
Tang et al.	$W = c_1(I_1 - 3) + c_2(I_2 - 3) + c_3\left(\exp\left(c_4(I_1 - 3)\right) - 1\right)$
Van Dam et al.	$W = c_1\left\{-\frac{1-c_2}{c_3^2}\left[(c_3 x + 1)\exp(-c_3 x) - 1\right] + \frac{1}{2}c_2 x^2\right\}$ with $x = \sqrt{c_4 I_1 + (1 - c_4) I_2 - 3}$

Chagnon *et al.*, *Hyperelastic Energy Densities for Soft Biological Tissues: A Review*, Journal of Elasticity (2015) 120

The passive model

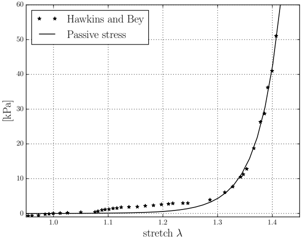
Ehret, Böl, Itskov, JMPS, 59 (2011)

$$W(\mathsf{C}) = \frac{\mu}{4}\left[\frac{1}{\alpha}(e^{\alpha(I_p-1)} - 1) + K_p - 1\right]$$

$$I_p = w_0 \frac{\operatorname{tr}\mathsf{C}}{3} + (1 - w_0)\operatorname{tr}\mathsf{CM}$$

$$K_p = w_0 \frac{\operatorname{tr}\mathsf{C}^{-1}}{3} + (1 - w_0)\operatorname{tr}\mathsf{C}^{-1}\mathsf{M}$$

$$\mathsf{C} = \mathsf{F}^T\mathsf{F}, \quad \det\mathsf{C} = 1, \quad \mathsf{M} = \mathbf{m} \otimes \mathbf{m}$$

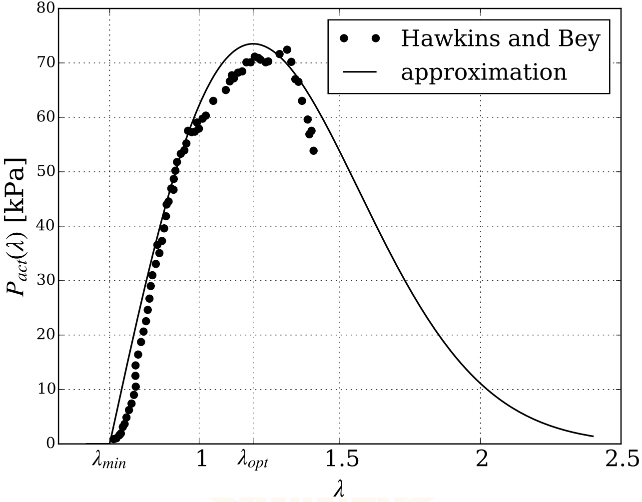


$\mu = 0.1599$ kPa
$\alpha = 19.35$
$w_0 = 0.7335$

The energy is polyconvex and coercive.
The parameters μ, α, w_0 can be fitted with experimental data on the relation between stress and strain in passive skeletal muscle

$$\mathsf{P} := \frac{\partial W}{\partial \mathsf{F}} - p\mathsf{F}^{-T} = 2\mathsf{F}\frac{\partial W}{\partial \mathsf{C}} - p\mathsf{F}^{-T}$$

Experimental data: the isometric tetanic contraction



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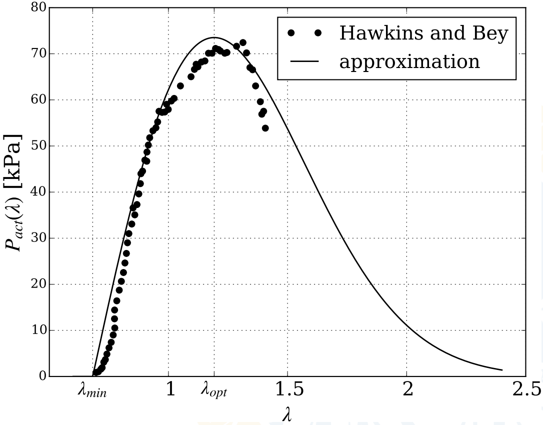
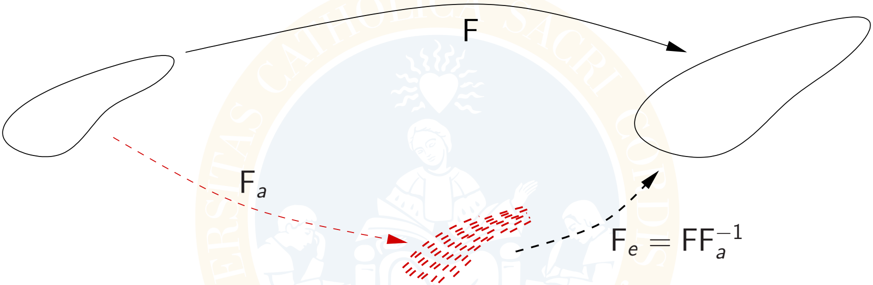


$\lambda_{min} = 0.682$
$\lambda_{opt} = 1.192$
$P_{opt} = 73.52\text{kPa}$

$$P_{act}(\lambda) = \begin{cases} P_{opt}\dfrac{\lambda_{min}-\lambda}{\lambda_{min}-\lambda_{opt}} \exp\left[\dfrac{(2\lambda_{min}-\lambda-\lambda_{opt})(\lambda-\lambda_{opt})}{2(\lambda_{min}-\lambda_{opt})^2}\right] & \text{if } \lambda > \lambda_{min} \\ 0 & \text{otherwise} \end{cases}$$

Activation

Active strain approach: multiplicative decomposition of the deformation gradient



$$F = F_e F_a \quad \Rightarrow \quad C_e = F_e^T F_e = F_a^{-T} C F_a^{-1}$$

$$\widehat{W}(C, F_a) = (\det F_a) W(C_e) = (\det F_a) W(F_a^{-T} C F_a^{-1}), \quad \det C = 1$$

Incompressible activation along the fibers

The muscle contracts along the fibers, hence we describe the activation by choosing

$$\mathsf{F}_a = (1-a)\mathbf{m}\otimes\mathbf{m} + \frac{1}{\sqrt{1-a}}(\mathsf{I}-\mathbf{m}\otimes\mathbf{m}).$$

Hence, only a scalar parameter $0 \leq a < 1$ has to be modeled.
Since $\det \mathsf{F}_a = 1$, it follows that

$$\widehat{W}(\mathsf{C}, a) = W(\mathsf{F}_a^{-T}\mathsf{C}\mathsf{F}_a^{-1})$$

$$\mathsf{F}_a^{-1} = \frac{1}{1-a}\mathbf{m}\otimes\mathbf{m} + \sqrt{1-a}(\mathsf{I}-\mathbf{m}\otimes\mathbf{m})$$

Uniaxial deformation

Let us consider an incompressible deformation along the fibers ($\mathbf{m} = \mathbf{e}_1$):

$$\mathsf{F} = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \frac{1}{\sqrt{\lambda}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{\lambda}} \end{pmatrix} \quad \Rightarrow \quad \mathsf{C}_e = \begin{pmatrix} \frac{\lambda^2}{(1-a)^2} & 0 & 0 \\ 0 & \frac{1-a}{\lambda} & 0 \\ 0 & 0 & \frac{1-a}{\lambda} \end{pmatrix}$$

so that $\widehat{W} = \widehat{W}(\lambda, a)$.

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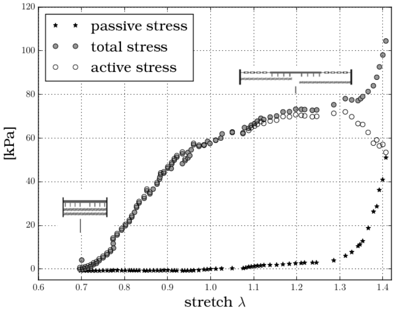

If we look at the experimental data, we can notice that the amount of active stress depend on the deformation. Hence it is customary to assume that a is a function of λ.

The active part of the stress

Considering $\widehat{W}(\lambda, a(\lambda))$, we can compute the stress along the fiber direction by

$$P_{tot}(\lambda, a(\lambda)) := \frac{\partial \widehat{W}}{\partial \lambda} + \frac{\partial \widehat{W}}{\partial a} a'.$$

In particular, the *passive stress* is given by $P_{pas}(\lambda) := P_{tot}(\lambda, 0)$ and the *active stress* by

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Solve for $a(\lambda)$:

$$\begin{cases} \dfrac{\partial \widehat{W}(\lambda, a(\lambda))}{\partial \lambda} + \dfrac{\partial \widehat{W}(\lambda, a(\lambda))}{\partial a} a'(\lambda) = P_{act}(\lambda) + P_{pas}(\lambda) \\ a(\lambda_{min}) = 0 \end{cases}$$

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$$I_e = \frac{w_0}{3}\,\mathrm{tr}(\mathrm{C}_e) + (1 - w_0)\,\mathrm{tr}(\mathrm{C}_e\mathrm{M}) = \left(1 - \frac{2}{3}w_0\right)\frac{\lambda^2}{(1 - a(\lambda))^2} + \frac{2}{3}w_0\frac{1 - a(\lambda)}{\lambda}$$

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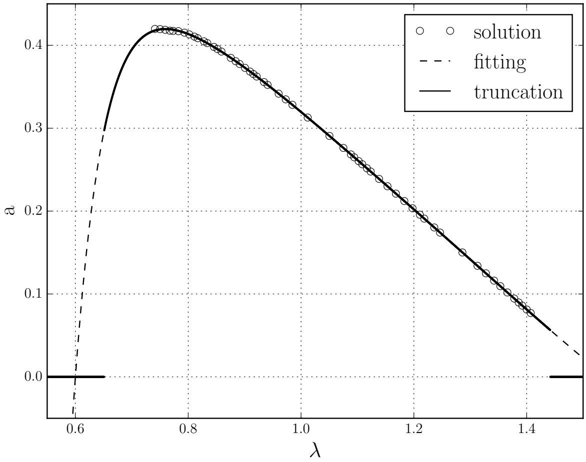
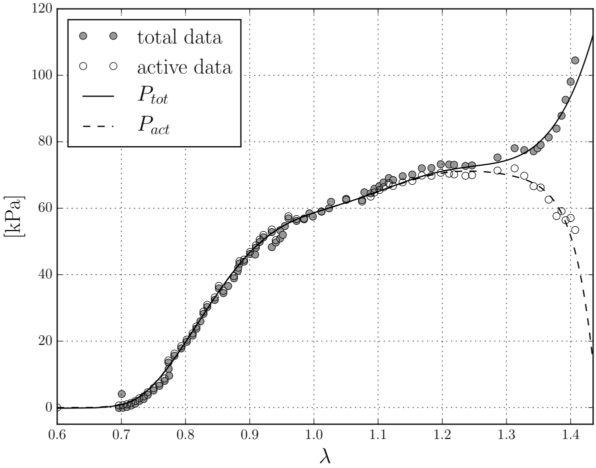
$$\boxed{a = a(\sqrt{\mathrm{tr}\,\mathrm{CM}})}$$

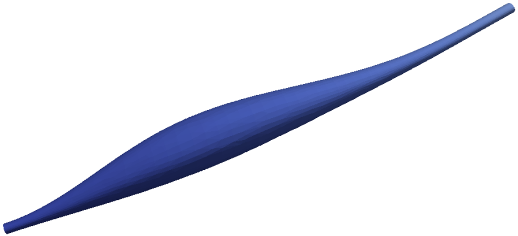
The numerical solution



The totat stress-strain relation



A “realistic” mesh



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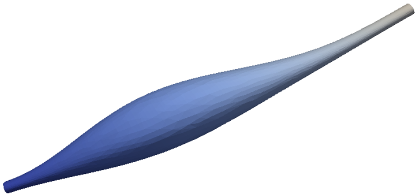

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