An Allen-Cahn theory of remodelling in fibre-reinforced biological tissues

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In the course of its life, a fibre-reinforced biological tissue may change its material properties in response to a variety of phenomena, such as mechanical or chemical processes, ageing, injuries, and degenerative diseases [1]. In the majority of cases, a fibre-reinforced tissue may be described as a fluid-saturated composite porous medium, in which the distribution and orientation of the collagen fibres tend to optimise the tissue's capability of bearing loads and conveying the fluid throughout its pore space. Since changes in the tissue's collagen content or in the pattern of fibre orientation may have consequences on the tissue's material behaviour, it is important to study how the fibres evolve. This, indeed, may help understand the mechanisms of deterioration of tissues and may provide guidelines for building engineered materials capable of replacing the original, damaged ones [1, 2].

Starting from some selected works on the subject, e.g., [1, 3, 4, 5, 6, 7], in the present contribution [8] we investigate the remodelling of a soft connective tissue with a statistically oriented network of collagen fibres. After assigning a suitable probability density distribution [9], we claim that the tissue's remodelling manifests itself through an evolution in time of the mean angle of orientation of the fibres, Q. The variability of such “remodelling parameter” is governed by a dynamic equation [5, 6, 7, 8], which allows for both a deformation-driven remodelling and an active one. The latter is here understood as the spontaneous attempt of the fibre network to go towards a histological pattern, i.e., the one optimising the tissue's properties. For the aforementioned purposes, the material is described as hyperelastic. Its strain energy, however, features a term of the Allen-Cahn type, which is introduced to account for the fibre remodelling. The proposed *remodelling energy* accounts for a non-trivial interaction with the deformation field, and depends on the gradient of Q, thereby taking explicitly into account its spatial resolution.

We show that, within the presented remodelling theory, the histological profile of Q in the tissue's undeformed configuration could be retrieved as a minimum of the remodelling energy, rather than by fitting experimental data. Afterwards, we discuss the numerical outcomes of a Finite Element benchmark of unconfined compression test on a sample of articular cartilage. As a consequence of a given loading protocol, the fibres rearrange within the tissue in a way to qualitatively resemble the pattern experimentally observed in a stressed and damaged tissue (i.e., the osteoarthritic one).

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