Proposed algorithm to evaluate $\log(1+x)$ for $|x| << 1$:

$$\mathrm{flt}\left(\frac{x\log(1+x)}{(1+x)-1}\right)$$

The residual transformation to which we are interested is given by

$$y \mapsto \frac{x\log y}{y-1} =: f(y)$$

where x has to be considered just a parameter; errors on x are already dealth with when computing the whole problem conditioning. The condition number is then obtained with the usual formula

$$K \approx \left|\frac{yf'(y)}{f(y)}\right|$$

Carrying out the computation, we get (after a few passages, also observe that the x simplifies)

$$K \approx \left|\frac{1-y+y\log y}{(y-1)\log y}\right|$$

and we are interested in its value for $y \approx 1$. It is then convenient to write $y = 1+\xi$ (I purposefully avoid to use the letter x in order to avoid confusion) and Taylor-expand around $\xi = 0$

$$K \approx \left|\frac{(\xi+1)(\xi-\frac{1}{2}\xi^2+\mathcal{O}(\xi^3))-\xi}{\xi(\xi+\mathcal{O}(\xi^2))}\right| = \left|\frac{\xi^2-\frac{1}{2}\xi^3+\xi-\frac{1}{2}\xi^2-\xi+\mathcal{O}(\xi^3)}{\xi^2+\mathcal{O}(\xi^3)}\right| \tag{1}$$

After simplification we finally obtain

$$K \approx \left|\frac{\frac{1}{2}+\mathcal{O}(\xi)}{1+\mathcal{O}(\xi)}\right| \to \frac{1}{2}$$

as $\xi \to 0$. So that we have a well-conditioned contribution even from this (delicate) residual transformation.

You should notice that it is essential that the two ξ in the numerator of (1) simplify. this cancellation is the main reason why the algorithm is actually stable.