② (degenerate bilinear form)

$$T = (v_i) \qquad S = (w_j)$$

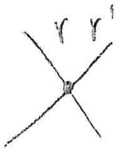
$$T \otimes S = (v_i v_j = b_{ij}) \qquad \in V^* \otimes V^* = \mathcal{T}_{02}$$

<u>Example</u> : a degenerate conic in $\mathbb{P}^2$

$\mathscr{C}$: $\underbrace{(\alpha_0 x_0 + \alpha_1 x_1 + \alpha_2 x_2)}_{r}\underbrace{(\beta_0 x_0 + \beta_1 x_1 + \beta_2 x_2)}_{r'} = 0$

$\parallel$

rr'



abuse of language : $rr' = 0$

$\rightsquigarrow$ the matrix $\hat{A} = (\alpha_i \beta_j) \quad i, j = 0, 1, 2$

can be rendered symmetric :

$$A = \left(\frac{\alpha_i \beta_j + \alpha_j \beta_i}{2} =: a_{ij} \right)$$

So that the equation of $\mathscr{C}$ reads : $\quad X^T A X = 0$

$A = A^T$

(<u>polarization</u> of a quadratic form)

③ $T = (a^i_j) \in V \otimes V^* \cong \mathrm{End}(V) \qquad T \in \mathcal{T}_{1,1} \quad S \in \mathcal{T}_{1,1}$

$S = (b^h_k) \in V \otimes V^* \qquad T \otimes S \in \mathcal{T}_{2,2}$

$T \otimes S = (a^i_j b^h_k)$

ex: $\begin{pmatrix} a^1_1 & a^1_2 \\ a^2_1 & a^2_2 \end{pmatrix} \otimes \begin{pmatrix} b^1_1 & b^1_2 \\ b^2_1 & b^2_2 \end{pmatrix}$

$$= \left(a^i_j \, b^h_k \right) \quad \text{(a } 4 \times 4 \text{ matrix)}$$

$\dim V = \dim V^* = 2$