QUASISTATIC LIMIT OF A DYNAMIC VISCOELASTIC MODEL WITH MEMORY

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1. Super short abstract

We study the behaviour of the solutions to a dynamic evolution problem for a viscoelastic model with long memory, when the rate of change of the data tends to zero. We prove that a suitably rescaled version of the solutions converges to the solution of the corresponding stationary problem.

2. Short abstract

In the work "Quasistatic limit of a dynamic viscoelastic model with memory" (published in Milan Journal of Mathematics 2021), in collaboration with G. Dal Maso, we study the problem of the quasistatic limit of Maxwell's model of viscoelasticity, that is

$$\ddot{u}(t) - \operatorname{div}((\mathbb{A}+\mathbb{B})eu(t)) + \int_{-\infty}^{t} \frac{1}{\beta} \mathrm{e}^{-\frac{t-\tau}{\beta}} \operatorname{div}(\mathbb{B}eu(\tau))\mathrm{d}\tau = f(t) \qquad \text{in } \Omega,\ t \in [0,T] \tag{2.1}$$

with some boundary and initial conditions (here eu represents the symmetric part of the gradient of the displacement u). By choosing a forcing term $f(t)$ of suitable regularity, we consider for every $\varepsilon > 0$ the solution $u^\varepsilon(t)$ to the viscoelastic problem (2.1) with forcing term $f(\varepsilon t)$, and then we consider the rescaled solution $u_\varepsilon(t) := u^\varepsilon(\frac{t}{\varepsilon})$, which is a solution to the following equation

$$\varepsilon^2 \ddot{u}_\varepsilon(t) - \operatorname{div}((\mathbb{A}+\mathbb{B})eu_\varepsilon(t)) + \int_{-\infty}^{t} \frac{1}{\beta\varepsilon} \mathrm{e}^{-\frac{t-\tau}{\beta\varepsilon}} \operatorname{div}(\mathbb{B}eu_\varepsilon(\tau))\mathrm{d}\tau = f(t) \qquad \text{in } \Omega,\ t \in [0,T].$$

We are able to show that $u_\varepsilon(t)$ converges locally uniformly and strongly in $L^2(0,T;H^1(\Omega))$, as $\varepsilon \to 0^+$, to the solution $u(t)$ of the following stationary problem

$$-\operatorname{div}(\mathbb{A}eu(t)) = f(t) \quad \text{in } \Omega,\ t \in [0,T].$$

If the compatibility condition is satisfied, then the convergence is uniformly on the whole interval $[0,T]$. Moreover, we prove that $\varepsilon\dot{u}_\varepsilon$ converges pointwise and strongly in $L^2(0,T;L^2(\Omega))$ to 0, as $\varepsilon \to 0^+$. The result is obtained first by stronger regularity assumptions on the data (by using the Laplace transform for functions with values in Hilbert spaces), and then by approximation and energy estimate we obtain the general result.

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