Two manifestations of the BKL decomposition: growth and multiscale remodelling

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Abstract

The mechanical response of a living tissue to internal or external stimuli leads, in general, to a rearrangement of its internal structure accompanying the deformation of the tissue [1]. In the literature, several examples can be found showing that the study of this kind of phenomena can be addressed by having recourse to the BKL (Bilby-Kröner-Lee) multiplicative decomposition [2, 3]. In particular, in this presentation, we focus on the use of the BKL decomposition to study growth and remodelling in biological tissues.
The talk is split into two parts. The first speech concerns the growth of a tumour in the avascular stage [4]. To account for the growth-induced structural changes of the tumour under study, we employ the BKL decomposition to introduce a "growth tensor", whose time evolution is strictly related to the mass transfer among the phases constituting the growing tumour tissue. The second part of the presentation focuses on the study of heterogeneous media with evolving micro-structure. In this case, the BKL decomposition is reformulated in a scale-dependent fashion and, by exploiting the information available at the smaller scale, the two-scale asymptotic homogenization technique is performed to obtain an effective description of the heterogeneous, remodelling medium [5].

References

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