

# Non-linear wave propagation and non-equilibrium thermodynamics

Tommaso Ruggeri  
Dipartimento di Matematica and  
Alma Mater Research Center on Applied Mathematics AM<sup>2</sup>  
Università di Bologna

In an endeavor to understand ubiquitous non-equilibrium phenomena, a number of thermodynamic theories have been proposed and developed. Among others, thermodynamics of irreversible processes (TIP) adopts the assumption of local equilibrium as one of the essential theoretical ingredients and has become popular, owing to its systematic and comprehensive theoretical structure. The Navier-Stokes Fourier (NSF) theory for fluids is its typical example. TIP has repeatedly demonstrated its practical usefulness in various situations. However, it is well known that TIP involves the problem of infinite speed of disturbances, which is sometimes called "the paradox of heat conduction", due to the parabolic character of the basic equations with spatially non-local constitutive equations.

In these lectures we start from the pioneer work of Carlo Cattaneo to present the modern approach of Rational Extended Thermodynamics (RET) that has the mesoscopic foundation in the kinetic theory and is described by differential symmetric hyperbolic systems of balance laws with local constitutive equations. The closure process for constitutive equations is discussed and in particular the intimate interplay between the entropy principle, the variational method of the Maximum Entropy Principle (MEP) and the symmetric hyperbolic systems is remarked. Non-linear wave propagation is discussed, with particular emphasis on shock waves where there is a perfect agreement between the theory and the experiments. Moreover, the lectures focus also on the qualitative analysis, proving the existence of global smooth solutions for sufficiently small initial data.

As the study of highly non-equilibrium phenomena becomes more and more important in modern technology like nano-technology, reenter in the atmosphere of satellites, aeronautical engineering, RET seems to be a promising theory.

[1] I. Müller, T. Ruggeri: *Rational Extended Thermodynamics*, Springer Tracts in Natural Philosophy 37 (II edition), Springer-Verlag, New York (1998).

[2] T. Ruggeri, M. Sugiyama: *Rational Extended Thermodynamics beyond the Monatomic Gas*, Springer, Cham, Heidelberg, New York, Dordrecht, London (2015).