Homogenized modeling for vascularized poroelastic tumors

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Abstract

A new mathematical model for the macroscopic behavior of a material composed of a poroelastic solid embedding a Newtonian fluid network phase (also referred to as vascularized poroelastic material), with fluid transport between them, is derived via asymptotic homogenization [4]. The typical distance between the vessels/channels (microscale) is much smaller than the average size of a whole domain (macroscale). The homogeneous and isotropic Biot's equation (in the quasi-static case and in absence of volume forces) for the poroelastic phase and the Stokes' problem for the fluid network are coupled through a fluid-structure interaction problem which accounts for fluid transport between the two phases; the latter is driven by the pressure difference between the two compartments. The averaging process results in a new system of partial differential equations that formally reads as a double poroelastic, globally mass conserving, model, together with a new constitutive relationship for the whole material which encodes the role of both pore and fluid network pressures. The mathematical model describes the mutual interplay among fluid filling the pores, flow in the network, transport between compartments, and linear elastic deformation of the (potentially compressible) elastic matrix comprising the poroelastic phase. Assuming periodicity at the microscale level, the model is computationally feasible, as it holds on the macroscale only (where the microstructure is smoothed out), and encodes geometrical information on the microvessels in its coefficients, which are to be computed solving classical periodic cell problems. Recently developed double porosity models are recovered when deformations of the elastic matrix are neglected. We present practical examples of application of the latter models [1, 2, 3] in the context of vascularized tumors and discuss how deformations of the matrix could affect the fluid and drug transport profile in the tissue.

References

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