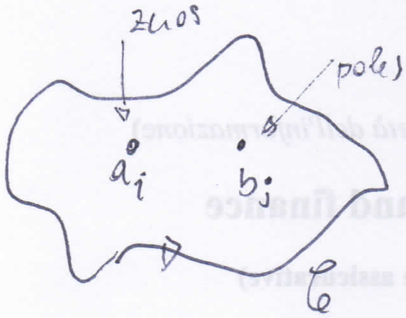
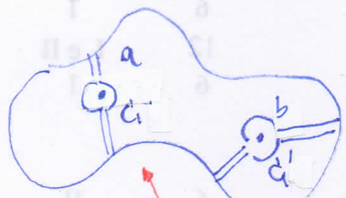
Addendum to Lecture XXVII-2-add



$$(\diamond)\quad \boxed{\frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{z f'(z)}{f(z)}\,dz = \sum a_i - \sum b_j}$$

Proof (sketch)

$$\frac{f'}{f} = \frac{\partial}{\partial z}\log f$$



$$\frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{z f'(z)}{f(z)}\,dz =$$

$$\frac{1}{2\pi i}\left[\sum \oint_{C_i} \frac{z f'(z)}{f(z)}\,dz + \sum \oint_{C_i'} \frac{z f'(z)}{f(z)}\,dz\right]$$

small circles around a & b

a: zero of mult. k $\quad f \sim (z-a)^k \quad f' \sim k(z-a)^{k-1} \quad z \sim a$

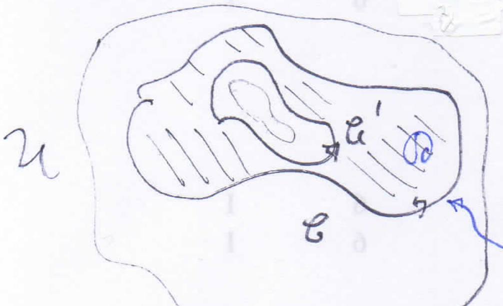
$$\frac{f'}{f} = \frac{k}{z-a} \Rightarrow \frac{1}{2\pi i}\oint_{C} \frac{z f'(z)}{f(z)}\,dz \to k\cdot a$$

Similarly $\quad f \sim (z-b)^{-k} \quad k>0$

b: pole $\quad f' \sim (-k)(z-b)^{-k-1}$

$$f'/f \sim -\frac{k}{z-b} \Rightarrow \frac{1}{2\pi i}\oint_{C'} z\frac{f'}{f}\,dz \to -k\cdot b$$

$\Rightarrow (\diamond)$ follows



basic principle: $g \in \mathcal{H}(U) \Rightarrow$

$$\int_{\mathcal{C}} g\,dz = \int_{\mathcal{C}'} g\,dz \quad \text{(Cauchy)} \qquad \mathcal{C} \sim \mathcal{C}'$$

(proved via Green, if g is assumed a priori to be $\mathcal{C}^1$; however, the result can be proved without this assumption)