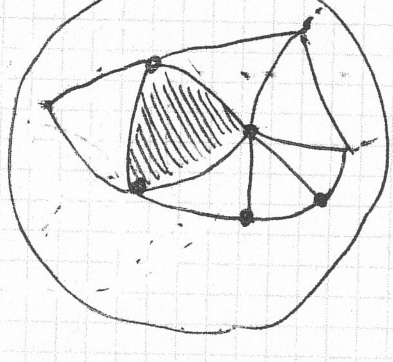
Aside

★ The Cauchy proof of

$$\underbrace{V}_{\text{vertices}} - \underbrace{E}_{\text{edges}} + \underbrace{F}_{\text{faces}} = 2$$

(for a sphere)

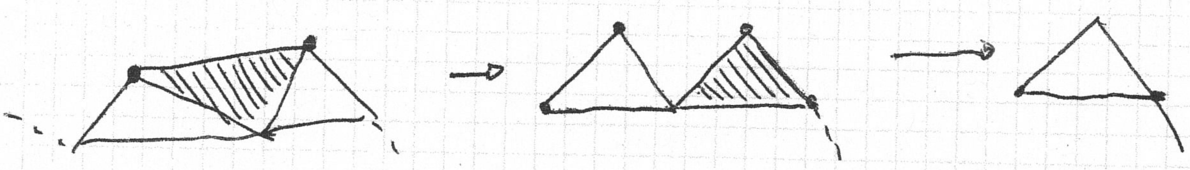


given any triangulation

Let $V - E + F = x$

After removing a triangle, the above sum becomes $y = x - 1$. Let us then show that $y = 1$.

Upon placing the surface on a plane (for instance via a suitable stereographic projection, we successively remove triangles, from the exterior to the interior



$y \longrightarrow y \qquad y$

The number y does not change throughout the process.

Eventually, we are left with a single triangle, for which $y = 1$ □