Therefore $\quad n\langle p_i\rangle \sim n\langle p_0\rangle \quad \forall n \in \mathbb{Z}$

$$\langle p_0\rangle \equiv \langle 0\rangle$$

$\Rightarrow$ any zero cycle is homologically equivalent to $n\langle 0\rangle$ for some $n \in \mathbb{Z}$, whence $\boxed{H_0(K) = \mathbb{Z}}$ with $K = \Delta$

<u>Let us compute $H_1(K)$.</u>

1-cycles: $\quad \partial\big(\, l\,\langle 0,1\rangle + m\,\langle 1,2\rangle + n\,\langle 2,0\rangle \,\big) = 0$, $\quad l, m, n \in \mathbb{Z}$

$$l(\langle 1\rangle - \langle 0\rangle) + m(\langle 2\rangle - \langle 1\rangle) + n(\langle 0\rangle - \langle 2\rangle) = 0$$

$$\Rightarrow \quad (n-l)\langle 0\rangle + (l-m)\langle 1\rangle + (m-n)\langle 2\rangle = 0$$

$$\Rightarrow \quad l = m = n$$

$\Rightarrow$ any 1-cycle is an integer multiple of

$$c = \langle 0,1\rangle + \langle 1,2\rangle + \langle 2,0\rangle$$

hence $\boxed{H_1(\mathbb{Z}) \cong \mathbb{Z}}$

Same exercise for $K =$  full

one gets

$$\boxed{H_*(K = \ ) = \begin{cases} \mathbb{Z} & p = 0 \\ 0 & p = 1 \\ 0 & p = 2 \\ 0 & p = 3 \end{cases}}$$

hint:

H_0 : same argument

H_1  any 1-cycle is the boundary of a 2-chain

H_2 : ...

H_3 : ...