GEOMETRIA II

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(1) Data la curva $\mathcal{C}$: $\begin{cases} x^2+y^2-z^2-4=0 & \Sigma \\ 2z+x-2=0 & \pi \end{cases}$ 

(cos'è?) calcolare la curvatura normale $k_n^{\mathcal{C}}(P)$, $P:(2,0,0)$ tramite il teorema di Meusnier.

Si determini poi $k^{\mathcal{C}}(P)$ (curvatura di $\mathcal{C}$ in P)

(2) Si calcoli nuovamente $k^{\mathcal{C}}(P)$ direttamente (calcolo esplicito) e si determini la curvatura geodetica $k_g^{\mathcal{C}}$ in P.

Tempo a disposizione: 1h30m

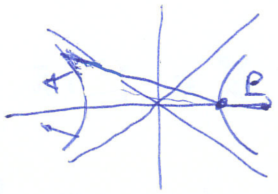
Le risposte vanno adeguatamente giustificate

Geo II   12/7/2018

① $\mathscr{C}: \begin{cases} x^2+y^2-z^2-4=0 \\ 2z+x-2=0 \end{cases}$ $P:(2,0,0)$

$z=0 \Rightarrow x=2$

Curvatura normale $k_n^{\mathscr{C}}(P)$



Meusnier: $k_n^{\mathscr{C}}(P) = k_n^{\mathscr{C}'}(P)$

$\mathscr{C}': \begin{cases} x^2+y^2-4=0 \\ z=0 \end{cases}$ $R=2$

$$k_n^{\mathscr{C}'}(P) = -\frac{1}{2} = k_n^{\mathscr{C}}(P)$$

Calcoliamo $k^{\mathscr{C}}(P)$

<u>1° modo</u>: diretto (calcolo implicito)

$\mathscr{C}: \underline{r} = \underline{r}(s)$ s: lunghezza d'arco

$\begin{cases} \cancel{2}xx' + \cancel{2}yy' - \cancel{2}zz' = 0 \\ 2z' + x' = 0 \end{cases}$

in P: $\begin{cases} 2x' = 0 \\ 2z' + x' = 0 \end{cases}$ $x' = 0$, $z' = 0$

$\begin{cases} x'^2 + xx'' + y'^2 + yy'' - z'^2 - zz'' = 0 \\ 2z'' + x'' = 0 \end{cases}$

in P: $2x'' + 1 = 0$

$2z'' + x'' = 0$

$x'' = -\frac{1}{2}$ $2z'' - \frac{1}{2} = 0$; $z'' = \frac{1}{4}$

$x'^2 + y'^2 + z'^2 = 1$ in P: $y' = \pm 1$ scelgo $+$

$\cancel{2}x'x'' + \cancel{2}y'y'' + \cancel{2}z'z'' = 0$ in P $y'' = 0$

$\underline{r} = (2, 0, 0)$

$\underline{r}' = (0, 1, 0)$

$\underline{r}'' = (-\frac{1}{2}, 0, \frac{1}{4})$

$$k^{\mathscr{C}}(P) = \|\underline{r}''(P)\| = \sqrt{\frac{1}{4} + \frac{1}{16}}$$

$$= \sqrt{\frac{4+1}{16}} = \sqrt{\frac{5}{16}} = \frac{\sqrt{5}}{4}$$

2° modo : utilizziamo $k_n^{\mathcal{C}}(P) = -\frac{1}{2}$

Ci serve $\langle n, N \rangle$ $\qquad N(P) = (1, 0, 0)$

attenzione

$$n = -\frac{1}{\sqrt{5}}(2, 0, 1)$$

$b \propto (1, 0, 2)$

$n \propto (2, 0, -1)$

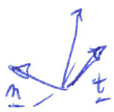
$$\langle n, N \rangle = -\frac{2}{\sqrt{5}}$$

$$k^{\mathcal{C}} = \frac{1}{\langle n, N \rangle} k_n^{\mathcal{C}} = -\frac{1}{2} \cdot \left(-\frac{\sqrt{5}}{2}\right) = +\frac{\sqrt{5}}{4}$$

Curvatura geodetica in P

$$|k_g^{\mathcal{C}}| = \sqrt{{k^{\mathcal{C}}}^2 - {k_n^{\mathcal{C}}}^2} = \sqrt{\frac{5}{16} - \frac{1}{4}} = \sqrt{\frac{5-4}{16}} = \frac{1}{4}$$

Segno: calcoliamo $\langle b, N \rangle$ $\qquad b = \frac{1}{\sqrt{5}}(1, 0, 2)$



$N = (1, 0, 0)$

$\langle b, N \rangle = \frac{1}{\sqrt{5}} > 0$

$$k_g^{\mathcal{C}} = +\frac{1}{4}$$

controllo: $k_g^{\mathcal{C}} = k^{\mathcal{C}} \langle b, N \rangle = \frac{\sqrt{5}}{4} \cdot \frac{1}{\sqrt{5}} = \frac{1}{4}$ ✓
