GEOMETRIA II

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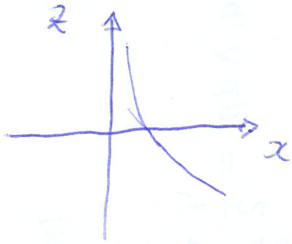


① Si consideri la curva $\mathcal{C}: \; z = -\log x$

e sia Σ la superficie ottenuta ruotando $\mathcal{C}$ attorno all'asse z, orientata come in figura. Si determinino eq. parametriche per Σ e successivamente si determinino le due forme fondamentali, K, H e le curvature principali in un pto generico.

② Con riferimento all'esercizio 1, si calcolino k_1, k_2, K per altra via.

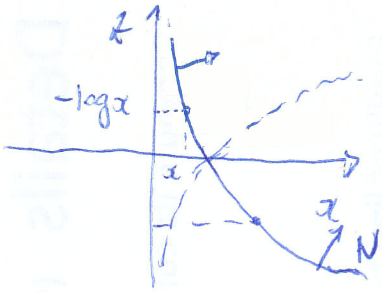
③ (fac.) Dire se esiste un parallelo di Σ sul quale $H \equiv 0$.

Tempo a disposizione: 1h 30m

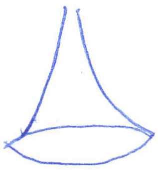
Le risposte vanno adeguatamente giustificate

$\boxed{\text{Geo II} \quad 6/6/18}$

1



$z = -\log x$ ra ruotata



$$\Sigma : \begin{cases} x = \rho \cos\varphi \\ y = \rho \sin\varphi \\ z = -\log\rho \end{cases}$$

$$\underline{r} = (\rho\cos\varphi, \ \rho\sin\varphi, \ -\log\rho)$$

$$\underline{r}_\rho = (\cos\varphi, \ \sin\varphi, \ -\tfrac{1}{\rho})$$

$$\underline{r}_\varphi = (-\rho\sin\varphi, \ \rho\cos\varphi, \ 0)$$

$$\underline{r}_{\varphi\rho} = \underline{r}_{\rho\varphi} = (-\sin\varphi, \ \cos\varphi, \ 0)$$

$$\underline{r}_{\rho\rho} = (0, \ 0, \ \tfrac{1}{\rho^2})$$

$$\underline{r}_{\varphi\varphi} = (-\rho\cos\varphi, \ -\rho\sin\varphi, \ 0)$$

$$\underline{N} = \frac{1}{\| \ \|} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \cos\varphi & \sin\varphi & -\frac{1}{\rho} \\ -\rho\sin\varphi & \rho\cos\varphi & 0 \end{vmatrix} = \left\{ \underline{i}(\cos\varphi) - \underline{j}(-\sin\varphi) + \underline{k}\cdot\rho \right\} \cdot \frac{1}{\| \ \|}$$

$$= \frac{1}{\sqrt{1+\rho^2}} (\cos\varphi, \ \sin\varphi, \ \rho) \qquad \text{(orientamento ok)}$$

I^a ff: $E = 1 + \frac{1}{\rho^2}$, $F = 0$, $G = \rho^2$

II^a ff: $e = \langle \underline{N}, \underline{r}_{\rho\rho} \rangle = \frac{1}{\rho\sqrt{1+\rho^2}}$, $f = 0$, $g = \frac{-\rho}{\sqrt{1+\rho^2}}$

$$K = \frac{eg - \overset{0}{f^2}}{EG - \underset{0}{F^2}} = \frac{\frac{1}{\rho\sqrt{1+\rho^2}} \cdot \frac{-\rho}{\sqrt{1+\rho^2}}}{\rho^2\left(1+\frac{1}{\rho^2}\right)} = \frac{-\frac{1}{1+\rho^2}}{1+\rho^2}$$

$$= -\frac{1}{(1+\rho^2)^2} = k_1 k_2$$

$$k_1 = \frac{e}{E} = \frac{\frac{1}{\rho\sqrt{1+\rho^2}}}{\frac{\rho^2+1}{\rho^2}} = \frac{1}{(1+\rho^2)^{\frac{3}{2}}} \cdot \rho$$

$$k_2 = \frac{g}{G} = \frac{-\rho}{\sqrt{1+\rho^2}} \cdot \frac{1}{\rho^2} = -\frac{1}{(1+\rho^2)^{\frac{1}{2}}\rho}$$

$$\left(k_1 k_2 = -\frac{1}{(1+\rho^2)^2} \quad \checkmark \right)$$

③ fac

$$H = \frac{1}{2}(k_1 + k_2) = \frac{1}{2}\left\{ \frac{\rho}{(1+\rho^2)^{3/2}} - \frac{1}{(1+\rho^2)^{\frac{1}{2}}\rho} \right\}$$

$$\frac{\rho}{(1+\rho^2)^{\frac{3}{2}}} = \frac{1}{(1+\rho^2)^{\frac{1}{2}}\rho}$$

$$\rho^2 = (1+\rho^2)$$

$$\Rightarrow \quad 1 = 0 \quad !!$$

Dunque $\nexists$ il parallelo cercato

② 2° modo : grannormale

k_1 : curv. (norm) meridiano (sono > 0)

$$z = -\log x$$

$$z' = -\frac{1}{x}$$

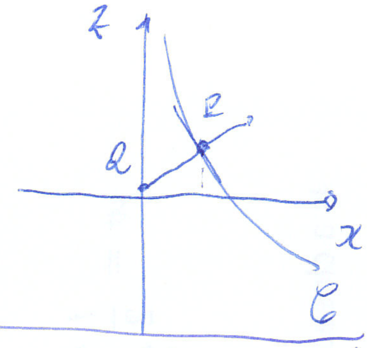
$$z'' = \frac{1}{x^2}$$

$$k_1 = \frac{\frac{1}{x^2}}{\left(1+\frac{1}{x^2}\right)^{3/2}}$$

$$= \frac{1}{x^2}\,\frac{(x^2)^{\frac{3}{2}}}{(1+x^2)^{3/2}} = \underbrace{\frac{x}{(1+x^2)^{3/2}}}_{\frac{\rho}{(1+\rho^2)^{3/2}}} \quad \checkmark$$

②

$\mathcal{R}_2$



calcoliamo $N = \overline{PQ}$

$z = -\log x$

retta per $P_0 : (x_0, \overbrace{-\log x_0}^{z_0})$

normale a τ :

$\mathcal{C}: \begin{cases} x = x \\ z = -\log x \end{cases}$ $\quad m = z' = -\frac{1}{x}$

$(m_\perp m = -1)$ $\quad m_\perp = x$

normale

$Q : \begin{cases} z - z_0 = x_0 (x - x_0) \\ x = 0 \quad \leftarrow \text{asse } z \end{cases}$

$$z = z_0 - x_0^2 = -\log x_0 - x_0^2$$

$Q : (0, z_0 - x_0^2)$

$P : (x_0, z_0)$

$$\overline{QP} = \sqrt{x_0^2 + x_0^4} = x_0 \sqrt{1 + x_0^2}$$

$$\mathcal{R}_2 = -\frac{1}{\overline{QP}} = -\frac{1}{x_0\sqrt{1+x_0^2}}$$

$$\mathcal{R}_2 = -\frac{1}{\rho(1+\rho^2)^{\frac{1}{2}}}$$

✓

Dunque, di nuovo

$$\mathcal{R}_1 = \frac{\rho}{(1+\rho^2)^{3/2}}$$

$$\mathcal{R}_2 = -\frac{1}{\rho(1+\rho^2)^{\frac{1}{2}}}$$

③ Non si ha mai $H = 0$