GEOMETRIA II

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Prova scritta del 26 settembre 2019

(1) Sia data la superficie $\Sigma: \; x^2+y^2-z^2-1=0$ con l'orientamento indicato nel punto $P:(0,-1,0)$ si determinino

la prima e la seconda forma fondamentale, nonché le curvature principali, la curvatura gaussiana, e la curvatura media, e l'operatore di forma. Si determinino poi le direzioni asintotiche e le linee asintotiche passanti per P

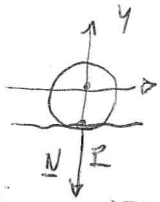
(2) Si determini l'evoluta della parabola $\mathcal{C}: \; y = 1 - x^2$ e se ne elabori il grafico

Tempo a disposizione: 1h30m

Le risposte vanno adeguatamente giustificate

① $\Sigma : f = x^2 + y^2 - z^2 - 1 = 0$

$P : (0, -1, 0)$



si può esplicitare $y = y(x,z) \equiv \varphi(x,z)$: infatti

$\nabla f = (2x, 2y, -2z) \qquad \nabla f|_P = (0, -2, 0)$

$\frac{\partial f}{\partial y} \neq 0 \Rightarrow$ (Dini) $y = \varphi(x,z)$

$$\underline{r} = (x, \underset{\varphi}{y}, z)$$

$$\underline{r}_x = (1, \varphi_x, 0)$$

$$\underline{r}_z = (0, \varphi_z, 1)$$

$$\underline{r}_{xx} = (0, \varphi_{xx}, 0)$$

$$\underline{r}_{xz} = (0, \varphi_{xz}, 0)$$

$$\underline{r}_{zz} = (0, \varphi_{zz}, 0)$$

in P: $\underline{N} = (0, -1, 0)$

$$x^2 + \varphi^2 - z^2 - 1 = 0$$

$\frac{\partial}{\partial x}$ $\quad \cancel{2}x + \cancel{2}\varphi\varphi_x = 0$

$\frac{\partial^2}{\partial x^2}$ $\quad 1 + \varphi_x^2 + \varphi\varphi_{xx} = 0$

$\frac{\partial^2}{\partial x \partial z}$ $\quad \varphi_z\varphi_x + \varphi\varphi_{xz} = 0$

$\frac{\partial}{\partial z}$ $\quad \cancel{2}\varphi\varphi_z - \cancel{2}z = 0$

$\frac{\partial^2}{\partial z^2}$ $\quad \varphi_z^2 + \varphi\varphi_{zz} - 1 = 0$

$\frac{\text{in } P}{\varphi = -1}$ $\qquad \varphi_x = 0 \qquad 1 + (-1)\varphi_{xx} = 0 \qquad \varphi_{xx} = +1$

$-\varphi_{xz} = 0 \qquad \varphi_{xz} = 0$

$-\varphi_z = 0 \qquad \varphi_z = 0 \qquad -\varphi_{zz} - 1 = 0 \qquad \varphi_{zz} = -1$

<u>in P</u>

$\underline{r} = (0, -1, 0)$

$\underline{r}_x = (1, 0, 0)$

$\underline{r}_y = (0 \quad 0 \quad 1)$

$\underline{r}_{xx} = (0, +1, 0)$

$\underline{r}_{xz} = (0, 0, 0)$

$\underline{r}_{zz} = (0, -1, 0)$

$\gamma \qquad N = (0, -1, 0)$

$E = 1 = G \qquad F = 0$

$e = -1 \qquad f = 0, \quad g = +1$

$$K = \frac{eg - f^2}{EG - F^2} = -1 \qquad H = \frac{1}{2}\,\frac{eG - 2Ff + Eg}{EG - F^2} =$$

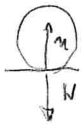
$$= \frac{1}{2}(-1 + 1) = 0$$

$k_1 = \frac{e}{E} = -1 \qquad k_2 = \frac{g}{G} = +1 \qquad \boxed{m(S) = \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}}$

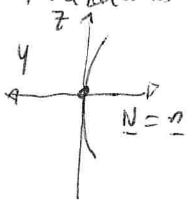
controllo

parallelo

$k_n = -1$



meridiano



$k_n = +1$

V

Direzioni asintotiche

in qui:

$$e\dot{u}^2 + 2\underset{0}{f}\dot{u}\dot{v} + g\dot{v}^2 = 0$$

$$-\dot{u}^2 + \dot{v}^2 = 0$$

$$\underset{\alpha}{\dot{u}} = \pm \underset{\beta}{\dot{v}}$$

Direzioni asintotiche.

rette:
$$P + \alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \pm \alpha \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 0 \\ \pm 1 \end{pmatrix} = \begin{pmatrix} \alpha \\ -1 \\ \pm\alpha \end{pmatrix}$$

ma queste sono anche linee asintotiche (regoli della sup.)

$$\begin{cases} x = \alpha \\ y = -1 \\ z = \pm\alpha \end{cases}$$

infatti

$$\alpha^2 + 1 - (\pm\alpha)^2 - 1 =$$
$$\alpha^2 + 1 - \alpha^2 - 1 = 0$$

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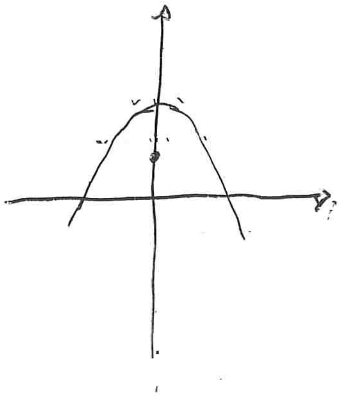
(2) Evoluta di $y = 1 - x^2$



inviluppo delle normali

$y' = -2x = m$

(coeff. ang della tang)

$m_\perp = -\frac{1}{m} = +\frac{1}{2x}$

normale in P_t

$\begin{cases} x = t \\ y = 1 - t^2 \end{cases}$

$y - (1 - t^2) = \frac{1}{2t}(x - t)$

$y - 1 + t^2 = \frac{1}{2t}x - \frac{1}{2}$

$2ty - 2t + 2t^3 - x + t = 0$

$$\boxed{2ty - x - t + 2t^3 = 0}$$

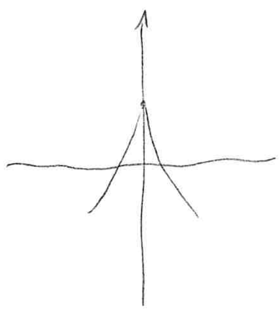
$\frac{\partial}{\partial t}$ $\quad 2y - 1 + 6t^2 = 0 \qquad y = \frac{1 - 6t^2}{2}$

$x = 2ty - t + 2t^3 = 2t\,\frac{1 - 6t^2}{2} - t + 2t^3$

$= (1 - 6t^2)t - t + 2t^3 = t - 6t^3 - t + 2t^3$

$= -4t^3$

$\begin{cases} x = -4t^3 \\ y = \frac{1 - 6t^2}{2} = \frac{1}{2} - 3t^2 \end{cases}$



$y = 0 \quad t = \pm\frac{1}{\sqrt{6}}$