GEOMETRIA II

Prof. M. Spera

Prova Scritta del 20 giugno 2019

① Sia data la curva $\mathcal{C}: \begin{cases} x^2+y^2-1=0 & \text{cilindro} \\ x+y+z=0 & \text{piano} \end{cases}$

[Cos'è ?] Determinare, in $P:(1,0,-1) \in \mathcal{C}$

$k^{\mathcal{C}}(P)$ e $\tau^{\mathcal{C}}(P)$. [Utilizzare il calcolo implicito]
curvatura torsione

② Con riferimento all'es. 1, determinare $k_g^{\mathcal{C}}(P)$ e $k_n^{\mathcal{C}}(P)$, leggendo $\mathcal{C}$ come curva di $\Sigma: x^2+y^2-1=0$
curv. geodetica curv. normale

Tempo a disposizione: 1h 30m

Le risposte vanno adeguatamente giustificate

Geo II 20/6/19

① $\mathcal{C}: \begin{cases} x^2+y^2-1=0 & \text{cilindro} \\ x+y+z=0 & \text{piano} \end{cases}$

$\mathcal{C}$ è un'ellisse sia $\boxed{P: (1,0,-1) \in \mathcal{C}}$

Calcoliamo $k(P)$ [$\tau \equiv 0$ poiché $\mathcal{C}$ è piana]

$x = x(s)$ ecc.

$$\begin{cases} \cancel{2}xx' + \cancel{2}yy' = 0 \\ x'+y'+z'=0 \end{cases}$$

$$x'^2+y'^2+z'^2=1$$

$$\cancel{2}x'x''+\cancel{2}y'y''+\cancel{2}z'z''=0$$

$$\begin{cases} x'^2+xx''+y'^2+yy''=0 \\ x''+y''+z''=0 \end{cases}$$

$$\boxed{k = \|\underline{r}''\|}$$

$1 \cdot x' = 0 \Rightarrow x'=0$

$y' = -z'$

$y'^2+z'^2=1$

$2y'^2=1$ $y' = \pm \frac{1}{\sqrt{2}}$

$\rightarrow$ scegliamo $+$ (la curvatura non cambia) $y' = +\frac{1}{\sqrt{2}}$

$\Rightarrow$ $r'(P) = \left(0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$$\begin{cases} 1 \cdot x'' + \frac{1}{2} = 0 \Rightarrow x'' = -\frac{1}{2} \\ x''+y''+z''=0 \end{cases}$$

$\cancel{\frac{1}{\sqrt{2}}}y'' - \cancel{\frac{1}{\sqrt{2}}}z''=0$

$y''-z''=0$

$y''=z''$

$\Rightarrow$ $-\frac{1}{2}+2y''=0$ $y'' = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

$\boxed{\underline{r}'' = \left(-\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)}$ ①

$\boxed{k =} \sqrt{\frac{1}{4}+\frac{1}{16}+\frac{1}{16}} = \sqrt{\frac{4+1+1}{16}}$

$= \sqrt{\frac{6}{16}} = \sqrt{\frac{3}{8}} = \boxed{\frac{1}{2}\sqrt{\frac{3}{2}}}$

② $\underline{r}'' = \underbrace{\frac{1}{2}\sqrt{\frac{3}{2}}}_{k} \cdot \underbrace{\frac{1}{k}\left(-\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)}_{\underline{n}}$ ← normale principale

$\underline{r}' = \left(0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

troviamo $\underline{b} = \pm \frac{1}{\sqrt{3}}(1, 1, 1) = \overset{\underline{t}}{\underline{r}'} \times \underline{n}$

determiniamo il segno corretto

$\underline{t} \times \underline{n} =$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2k} & \frac{1}{4k} & \frac{1}{4k} \end{vmatrix} = \underline{i}\left(\underbrace{+\frac{1}{4k}\frac{1}{\sqrt{2}} + \frac{1}{4k}\frac{1}{\sqrt{2}}}_{\frac{1}{\sqrt{3}}}\right) - \underline{j}\left(\underbrace{-\frac{1}{2k}\frac{1}{\sqrt{2}}}_{\frac{1}{\sqrt{3}}}\right) + \underline{k}\left(\underbrace{+\frac{1}{2k}\frac{1}{\sqrt{2}}}_{\frac{1}{\sqrt{3}}}\right)$$

$+\frac{1}{\sqrt{3}} = +\frac{1}{2\frac{1}{2}\sqrt{\frac{3}{2}}}\frac{1}{\sqrt{2}}$

$= +\frac{1}{4\frac{1}{2}\sqrt{\frac{3}{2}}}\frac{1}{\sqrt{2}} + \frac{1}{4\frac{1}{2}\sqrt{\frac{3}{2}}}\frac{1}{\sqrt{2}}$

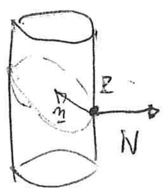
$= +\frac{1}{2\sqrt{3}} + \frac{1}{2\sqrt{3}} = +\frac{1}{\sqrt{3}}$

$\Rightarrow \boxed{\underline{b} = +\frac{1}{\sqrt{3}}(1,1,1)}$

Calcoliamo $\underline{N}(P)$

$P: (1, 0, -1)$

$f: x^2 + y^2 - 1 = 0$



$\nabla f: (2x, 2y, 0)$

$\nabla f(P) = (2, 0, 0) \qquad \underline{N} = (1, 0, 0)$ (or. corretto)

$k_g = \underset{\frac{1}{2}\sqrt{\frac{3}{2}}}{\underset{\parallel}{k}} \cdot \langle \underbrace{\underline{b}, \underline{N}}_{\frac{1}{\sqrt{3}}} \rangle = \frac{1}{2}\sqrt{\frac{3}{2}} \cdot \frac{1}{\sqrt{3}} = \frac{1}{2}\frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{1}{\sqrt{8}}$

$$k_n = k \cdot \langle \underline{n}, \underline{N} \rangle = \langle k\underline{n}, N \rangle = \langle r'', N \rangle = -\frac{1}{2}$$

Controllo: $k^2 = k_g^2 + k_n^2$

$$\frac{1}{8} + \frac{1}{4} = \frac{1+2}{8} = \frac{3}{8} = k^2 \qquad \checkmark$$